A Feasibility Study of a Hydromagnetic Waterjet Propulsion System

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This paper presents an analytical study for the determination of the performance characteristics of a proposed underwater waterjet propulsion system. The system is powered by the viscous drag pumping action of spherical micron-sized ferromagnetic particles accelerated by an external magnetic field. Some preliminary geometry details and thrust characteristics of a small system using a superconducting magnet and associated hardware are presented. The thrust/weight ratio of the proposed system is shown to be much less than one over the range of design parameters considered, placing serious limitations on the use of such a system.

Nomenclature

B = magnetic field intensity
C_p = particle concentration/unit volume
d_p = particle diameter
D = vehicle drag
f_m = magnetic force
g_c = gravitational constant
L = duct length
m_p = single particle mass
$\dot{m}_p$ = particle mass flow rate
N = particle number
NEF = net effectiveness factor
OEF = over-all effectiveness factor
p = pressure
P = power
Q = volumetric flow rate
r = radial coordinate
r_s = solenoid radius
R = duct radius
u = velocity
U_{in} = inlet fluid velocity
V = velocity
V_0 = vehicle speed
V_p = radial velocity of particle
V_r = velocity ratio
z = axial coordinate
β = length to diameter ratio of magnet
η_c = force conversion factor
η_{cc} = energy conversion coefficient
η_I = inlet efficiency
η_N = nozzle efficiency
η_p = propulsion efficiency
μ = viscosity
ρ = density

Subscripts

f = fluid
p = particle
r = radial component
z = axial component

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1. Introduction

WATERJET propulsion systems have received attention in recent years as possible marine craft powerplants. Hatte and Davis[1] discussed their application to hydrofoil craft and Arcand and Comolli[2] have considered waterjets as a propulsion power source for high-speed ships. The system to be discussed and analyzed in this paper is basically a waterjet propulsion powered one, however, the driving force for the jet flow of ambient sea water is to be provided by an externally applied magnetic field acting on small particles injected into the fluid. In many respects, except for the working fluid and specific propulsion application, this study represents an extension of the work of Hassel,[3,4] who investigated the possibility of generating thrust forces through viscous drag interaction between aerosol suspensions of micron-sized particles and the air through which they were accelerated.

In the proposed system, micron-sized particles of a ferromagnetic material are mixed with the sea water ducted into the vehicle. The magnetic field accelerates the particles which transfer their momentum to the fluid through the action of viscosity. After passing through the magnetic field, the fluid, which will have attained a higher pressure, is then discharged through a nozzle in the form of a jet, delivering a net positive thrust to the vehicle.

Unlike the case of a propulsion system with a limited amount of energy on board, this system could be looked upon as essentially a power limited system; however, it is also unlike a nuclear energy operated system in that the period of availability of a (superconducting) magnetic field and the supply of particles will determine a shorter duration for the mission. The acceleration of a given particle depends upon its surface area and the gradients in the applied magnetic field as well as other material properties. Also, the transfer of momentum between the particles and the water depends upon the contact surface area. For a given size of particles, the momentum transfer that can be effected with a given magnetic field is proportional to the number density or concentration of particles. As shown in Fig. 1, unless the particles are recycled, there is a limit to the amount of particles which one can employ. For a given number density of particles, the duration of the mission is proportional to the mass of particles which can be carried, unless the particles are collected and recirculated with a very low particle loss factor. Therefore, if no particle collection scheme is incorporated, this is both a power and energy limited system.

As previously mentioned, earlier investigations of Hassel[3,4] conducted with a similar scheme using atmospheric air as

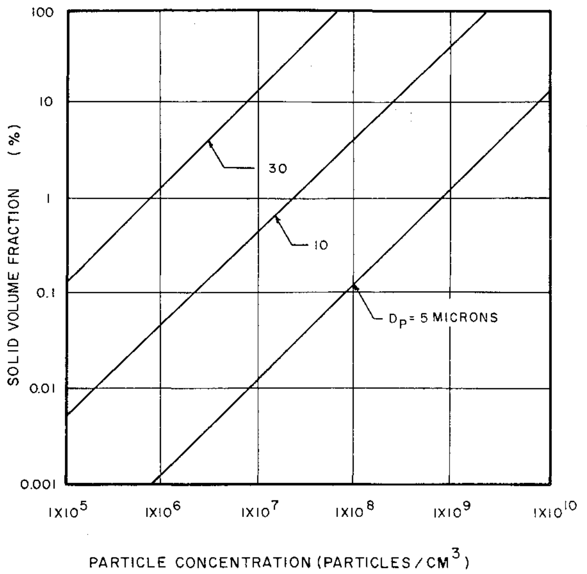

Fig. 1 Solid volume fraction as a function of particle size and concentration.

the working fluid have indicated that the ratio of the density of the fluid to the density of the suspended particles is an important factor in any practical application of the system. The objective of the investigation presented here is a preliminary assessment of the suitability of such a scheme of fluid acceleration for underwater or deep sea submersible jet propulsion using the ambient sea water as the propulsive fluid.

1.1 Proposed System

A schematic diagram of a design for this proposed hydromagnetic waterjet propulsion system is shown in Fig. 2. The system is comprised of three primary elements: the inlet system, the hydromagnetic pumping section, and the outlet section. It should be noted that the vertical section immediately after the inlet is included for illustration purposes to show that all the components need not be in the same horizontal plane. In practice, the design of the inlet and exhaust systems would be of critical importance to the over-all design,[1] however, this study will primarily be concerned with the analysis of the hydromagnetic pumping section. Sea water is drawn into the vessel at Sec. 00; it is mixed with solid particles which are injected at Sec. 22 and the two-phase fluid with the particle suspension is subjected to the action of a magnetic field between Secs. 33 and 44 and exists finally in the form of a jet at Sec. 66. The transfer of momentum to the fluid from any individual particle will cease, obviously, when it comes to rest; this may occur within the magnetic field as at Sec. cc in Fig. 2. If a large fraction of the initially injected particles can be collected at Sec. cc, the fluid ejected is mostly the original sea water. If it were not important to collect the particles, the operation of the vessel would become time-limited by the supply of particles and the duration over which the magnet will remain charged. However, with a superconducting magnet, there is essentially no such immediate limitation in view of the possibility of natural recharging; this may be one of the major advantages of the system.

It will be observed that no pump is included in Fig. 2. If a pump is installed in the system, between Secs. 11 and 22, it will not only provide an additional means of control but also, some new possibilities such as the injection of a hull boundary-layer control fluid and the application of the scheme to a hydrofoil waterjet may become possible. The pump of course needs a drive and its efficiency would be involved in the calculation of the over-all effectiveness of such a system.

2. General Systems Analysis

The choice of a propulsion system for a given vehicle depends upon the environment, the mission, the relationship of the most common operating point to the extremes of operating conditions and the economics of capital and operating costs as well as a quantitative estimation of reliability and safety factors in relation to the mission objectives. If a system were to be designed for a deep sea submersible, there is need for a morphological analysis of these related design factors, such has been done for many aeronautical systems.

A number of different morphological versions of the basic thrustor under discussion can be generated with reference to the following: 1) the disposition of the inlet, ducting, magnet, and nozzle relative to each other and the hull configuration, 2) the integration of the propulsor with the hull internally and externally, 3) the method of particle injection and the application of the magnetic field, and 4) the means of recovering the particles.

With regard to item, several alternative schemes are feasible other than the one shown in Fig. 2. For example, the inclusion of a pump will remove the need for a minimum length Secs. between 00 and 22; or the magnet may be set up in the vertical rather than the horizontal position. Several schemes will also result from an examination of item 2. It is possible, for example, to bring the ducting for water inside the hull only at the section where the magnetic field is present. The particle injection and the application of the magnetic field may be continuous or discontinuous with some scheme for the synchronization of the two. Such a nonsteady device may lead to the development of a form of pulse jet which could offer special advantages in the design of particle collectors. In the analysis to follow, however, the system will be analyzed using a steady magnetic field. The immediate advantages for this choice based on propulsion efficiency and system considerations were summarized by Hassel.[4]

A desired change in the performance of a given system can be obtained by adjusting any of the following parameters: 1) the volume rate of water flow, 2) the number density of injected particles, 3) the nozzle characteristics, and 4) the applied magnetic field. Other variables in the system may possibly be considered, but their variation may be unrealistic from the point of view of ease of operation and reliability.

In optimizing the system for a submersible with a given hull and speed-depth map, one is primarily interested in obtaining the highest hydrodynamic efficiency with ability to maneuver in all six degrees of freedom and with a high reliability for a weight consistent with buoyancy requirements. A complete optimization study of the present system would contain a determination of the magnetic field distribution which provide the largest increase in the pressure of water, that is, the highest "pumping" efficiency for a given fluid velocity and particle number density distribution. It is,

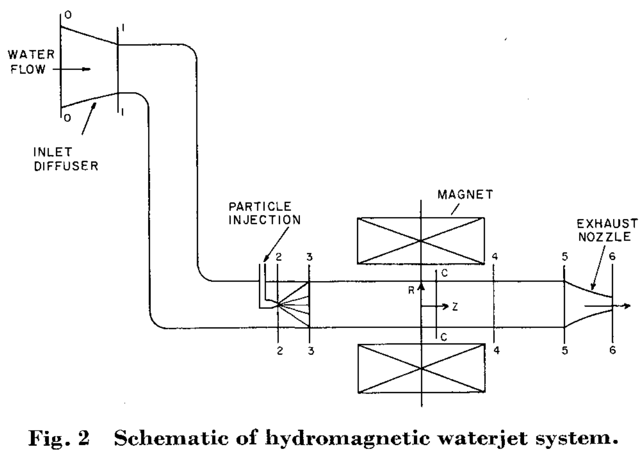

Fig. 2 Schematic of hydromagnetic waterjet system.

of course, possible to fix the magnet in view of space and weight considerations and determine the fluid or the particle velocity distribution with the same end in view.

It is difficult to define an efficiency for a device of this type. First of all, one may express the propulsive efficiency for the submersible as

$$P = DV_0/[\tfrac{1}{2}(\rho_f Q_f/g_c)(V_6^2 - V_0^2)] = DV_0^3/[\tfrac{1}{2}(\rho_f Q_f/g_c)(V_r^2 - 1)] \quad (1)$$

where D is the drag of the vehicle at speed V_0, ρ_f, and Q_f are the density and volumetric flow rate of the sea water, and V_r is the velocity ratio of the propulsor.

The propulsive efficiency itself should be examined in conjunction with an energy conversion coefficient defined by

$$\eta_{cc} = \text{(work done on the fluid/sec)/(power equivalent of the magnet charging cycle, the collection of particles and cryogenic fluid utilization)}$$

$$= \Delta p \cdot Q_f/(P_M + P_C + P_{CR}) \quad (2)$$

where Δp is the increase in pressure arising due to the momentum transfer from the particles to the fluid and the subscripts to P indicate the power equivalent of the three quantities mentioned in Eq. (2). Obviously, there may be considerable ambiguity in estimating the equivalent power input terms. It should be noted that, in satisfying conservation of energy requirements, the power expended in the collection of particles or the work rate done against the magnetic forces in order to remove the particles will be at least equal to the particle work rate on the fluid. Thus, in its simplest form, neglecting all other losses, this system is an energy conversion system that converts the mechanical collecting energy into fluid propulsion energy.

An "over-all effectiveness factor" of the propulsor section may be defined as

$$\text{OEF} = \eta_{cc}\eta_p \quad (3)$$

where η_p is the propulsion efficiency of the system. The uncertainties in the comparison of two systems on the basis of OEF will be in 1) the efficiency of conversion of the magnetic field force into impulse of the fluid and 2) the cutoff limits for the continuous operation of the system under different conditions.

In order to account for the contribution from the magnetic field, one may consider, for instance, a configuration with $N(r,z)$ particles across the Sec. at 33 in Fig. 2 with each particle acted on by a force $F_M(r,z)$ because of the magnetic field. If the length of the duct from 33 to cc is L and the velocity of the fluid at any point is $V_f(r,z)$, then the change in the impulse of the fluid over the length will be

$$2\pi\rho_f \int_0^R \int_0^L d(ru_f^2)drdz = 2\pi \int_0^R \int_0^L d\left[\frac{N(r,z)\cdot F_M(r,z)}{\eta_c}\right] drdz \quad (4)$$

where $R(z)$ is the radius of the duct and η_c is a force conversion factor. The difficulty in determining η_c is the same as that experienced in determining η_{cc}.

A "net effectiveness factor" for the system may be defined as

$$\text{NEF} = \eta_I \cdot \eta_N \cdot (\text{OEF}) \quad (5)$$

where η_I and η_N are the inlet efficiency (from Sec. 00 to Sec. 22) and the nozzle efficiency. A comparison of different systems can be made based upon the NEF 1) at the design point, 2) on an integrated basis over a specified mission, or 3) on a weighted mean performance over the speed map of the submersible.

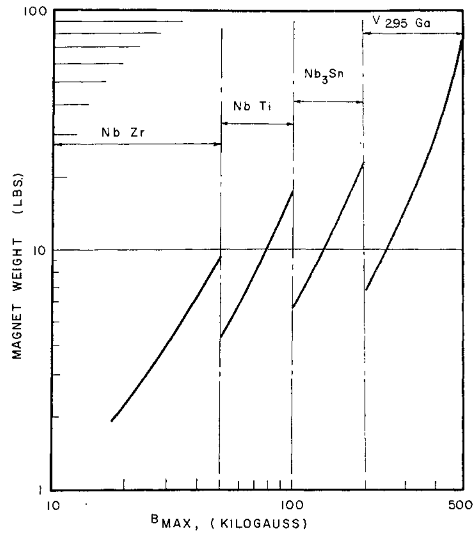

Fig. 3 Effect of material and maximum magnetic field on superconducting magnet weight.

3. Major System Components

3.1 Magnet and Associated Hardware

The magnet and its associated support systems form the most important part of the over-all thruster system and will have to be studied in detail before any actual device could be constructed. In this section of the paper, possible magnetic field sources will be discussed and some preliminary component weight and volume data will be presented.

As will be shown in the theoretical analysis, the most effective propulsor systems are those with an intense magnetic field operating over a small volume. It should also be noted that the shape of the magnetic field, determined by its geometrical configuration, can also affect the system's performance.

However, it was not the intent of this paper to present a completely "optimized" design for the system and the magnet. Therefore, the configurations analyzed in this paper were limited to solenoids of uniform inside and outside diameter. Furthermore, it was assumed that the magnets considered had a fixed ratio of magnet length to inner diameter, β, equal to 5. The effect of a variation in this ratio has been considered and has been found to be of minor importance compared with the other variables influencing the thrust output of the system.

Because of the high magnetic field (greater than 50 kgauss) required for efficient operation of the system, the use of conventional water-cooled magnets for the generation of these intense magnetic fields would be prohibitive in a deep sea submersible. To illustrate this point, according to Bitter,[5] a magnet of this type producing a maximum field of approximately 125 kgauss in a solenoid of radius 1.43 cm and 5.72 cm long would require a power input of 1.88 Mw and in addition would require a cooling water flow of 350 gpm.

The use of a superconducting solenoid for this application has been assumed, and with today's rapidly expanding superconductor technology this assumption will certainly become more justified and practical. On the negative side, superconducting solenoids are complicated by the requirement of maintaining the magnet windings at liquid helium temperatures; however, recent advances in cryogenics[6] have solved many of the associated cooling problems. For a large magnet such a cryogenic requirement might require the use of complex refrigeration systems. For the relatively small solenoid systems considered in this paper, one might operate the system for a number of hours by charging the insulated magnet container from a liquid helium source. The latter

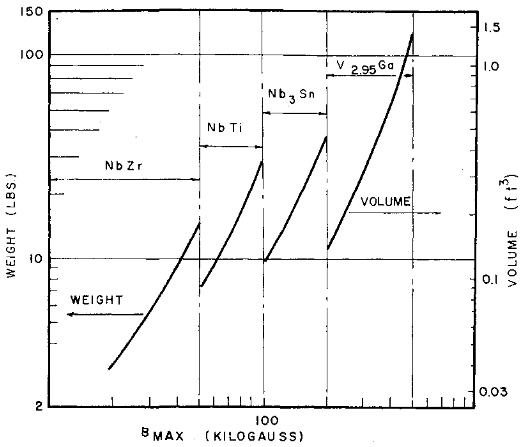

Fig. 4 Effect of material and maximum magnetic field on total magnet system weight and volume.

method of cooling has been assumed for the propulsor system, and since such systems have already been built for use in rocket flight tests,[7] no serious technical problems in insulated container systems are anticipated.

The specification of the exact superconductor material and solenoid configuration is quite an involved process, however, there are many excellent numerical techniques available for the specification of the magnet geometry once the material type and current density are determined.[8] The superconducting material choice is important since the produced field is limited due to the fact that the field component perpendicular to the current cannot exceed a critical value above which the superconducting properties of the material are destroyed. In order to avoid an involved study of the properties of superconducting materials, assumptions were made for the maximum field, current, and magnet weight density of the commonly used alloy superconductors (Nb-Zr, Nb-Ti, and Nb_3Sn) as well as a proposed alloy $V_{2.95}Ga$. These assumptions are summarized in Table 1 and are based on data extrapolated from Refs. 9–12. Using a solenoid inner radius of 1 cm, calculations for the magnet weight (and volume) as a function of material type and maximum field were carried out and are shown in Fig. 3.

The calculation of the insulated magnet container or dewar weight and volume is influenced by the current state-of-the-art and depends primarily on the method of insulation. Once the method of insulation and dewar construction are specified the volume (or weight) of the magnet container system can be calculated from the sum of its individual parts which include: magnet volume, liquid helium needed for magnet cool-down, liquid helium needed for heat losses, and container plus insulator volume. It was assumed that the first two terms would predominate and that, for a conservative estimate, 0.5 liters of helium/lb of magnet weight would be needed for cooldown.[11] Based on these assumptions, and the previous magnet weight calculations, it is possible to calculate the system volume and, in turn, the total system weight. The latter is accomplished with the estimate (for conventional dewars) that the container weight density is 17 lb/ft^3.[13] Figure 4 summarizes the results of the calculations for magnet system weight and volume.

3.2 Particle Injection and Collection Scheme

The performance of the hydromagnetic pumping section as a fluid accelerator depends entirely on the acceleration imparted to the particles between the injection and collection sections. It can be seen in Fig. 2, that after the particles are injected into the fluid at position 22, they diffuse into the stream and are affected by the magnetic field between the stations 33 and 44. Position 44, defined as the equilibration length, is the point where the velocity of a particle remaining on the centerline could reach zero, reverse its trajectory and then yield a negative thrust contribution. However, due to a radial force field within the solenoid there is a tendency for the particles to migrate towards the walls where they may be collected (or adhere to the walls). Due to the radial velocity gradient in the fluid, a particle at the flow centerline is in an unstable position, thus all particles will eventually end up near the flow channel walls. Based on the analytical model to be discussed, sample calculations have shown that all particles will contribute a positive net thrust to the system. Depending on various system design variables to be discussed and the collection scheme at Sec. cc, there is also the possibility that a large number of particles may be swept out of the pumping section. Thus, the collection scheme at a point past (or at) the center of the magnet has to consider the initial particle profile and fluid velocity as typical design parameters as well as the need to reuse the collected particles. Due to the geometry of the system, the collector design is undoubtedly a difficult task. The injection scheme, although possibly easier to design, is critical to the whole process of interaction between the magnetic field, fluid, and the particles.

The state of the particles at any section of the duct after injection may be described in terms of the size and number density distributions across the section of the duct and the velocity distribution of the particles relative to the fluid. Agglomeration and disintegration processes may further complicate the problem, but these are considered to be negligible effects in the present analysis. As previously mentioned, the particle distribution parameters are of fundamental significance at Sec. 33 where the particles first sense the action of the magnetic field. The state of the particles at Sec. 33 is a function of 1) the location of particle injection relative to the magnet, that is, the location of Sec. 22, 2) the fluid velocity distribution at Sec. 22, 3) the time dependence of the injection process, and 4) the duct boundary effects between Secs. 22 and 33. It is interesting to note that it may be feasible to include several sources of particles at Sec. 22. It is possible that there may be some further influence on the state of the particles at Sec. 33 and downstream due to the stratifications in the fluid and the gravitational field, but these effects are assumed to be small.

In carrying out an analysis of the diffusion of a continuously injected stream of particles into a moving fluid, one should consider 1) the particle parameters and 2) the diffusion mechanism. Among the particle parameters, namely the shape, size and number density distributions, and the mass fraction (per unit volume and per unit area of cross section of duct), it is necessary to have at least some knowledge of the size distribution to provide a mean size.

The simplest means of injecting the particles is by fluidizing them and injecting the slurry into the main flow stream. It is important that the slurry does not clog the injector. In view of the fact that the thickness of the slurry is a measure of the amount of particles injected, some method of continually assessing the slurry should be included in the control of the injector. This may be achieved by means of a simple meter operating on the basis of transmitted light through the slurry.

Table 1 Assumed values of magnet parameters for various superconducting alloys

Material	Maximum magnetic field, kgauss	Maximum current density, amp/cm^3	Magnet weight density, lb/cm^3
Nb-Zr	50	1.5×10^4	20
Nb-Ti	100	2.0×10^4	16
Nb_3Sn	200	3.0×10^4	12
$V_{2.95}Ga$	500	5.0×10^4	10

An efficient collector system is needed so that the weight of the particles carried will be a minimum. Calculated results for the total particle flow rate through the inlet injector for typical values of particle size and concentration are given in Fig. 5. As previously mentioned, there is a natural tendency for the particles to migrate towards the wall as they are drawn into the center of the magnet, thus facilitating particle collection. This trend for a range of inlet particle profiles is shown in Fig. 6 and demonstrates that by a combination of duct geometry, magnet field distribution, and particle number density, it is possible to cause the major portion of the particles to accumulate at the outer radius of the duct. One possible collection scheme, as first suggested by Hassel[4] is to collect the particles by means of a moving belt to which the particles will adhere on impact. Certainly, an important part of any further design studies would be a thorough investigation of particle collection methods.

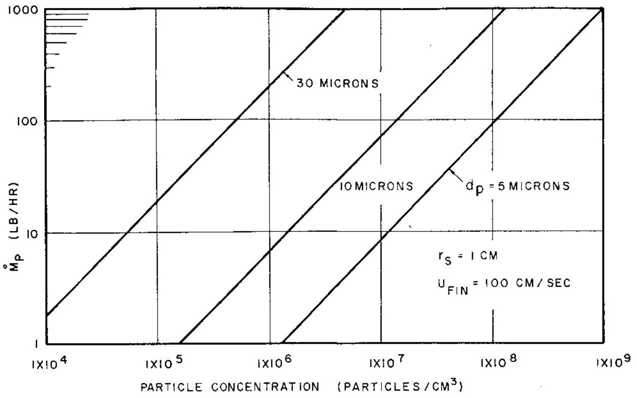

Fig. 5 Total particle flow rate as a function of inlet particle concentration and particle diameter.

4. Performance Estimation

4.1 Analytical Model

A complete theoretical model for a thrustor performance estimation may be obtained by analyzing 1) the action of a magnetic field on a two phase fluid flow with the particles as a discrete phase and 2) the inefficiencies involved in the generator of thrust. The analysis to be presented in this section will only consider momentum transfer in the hydromagnet pump section and will assume that the inefficiencies in converting the increased momentum of the fluid into useful thrust are negligible. As shown in Fig. 2, the r-z coordinate system has its origin in the center of the magnet and the control volume to be studied is bounded by the walls of the duct and Secs. 33 to cc.

4.2 Hydromagnetics of Two-Phase Flow

The fluid as well as the particles are considered incompressible and the inter-particle attractive forces are neglected. It is assumed that the particles are sufficiently large so that Brownian motion can be neglected and that they stay in suspension throughout the control volume. (Unless they come to rest on the wall of the flow duct.) Furthermore, the analysis to be summarized in this section is based on the following additional assumptions and is a simplified version of previous work by Hwang, McGowan, and Murthy[14]: 1) The fluid which flows in a constant diameter duct with negligibly small wall friction losses is assumed to be nonconducting. 2) The ferromagnetic particles in suspension are spherical in shape, therefore a single characteristic dimension will suffice for all calculations. They have a finite electrical conductivity and are small enough to neglect induced fields. The interaction between the particles and the fluid is governed only by the viscosity of the fluid and the number density distribution of particles, and there are no field gradients of temperature or pressure. 3) The applied magnetic field is symmetric about the r-z coordinates and is completely described by the distribution of the intensity of the field and the associated gradients in the axial and radial directions.

Table 2 Assumptions and specifications used in analysis of hydromagnetic pumping section

Centerline inlet fluid velocity, cm/sec	10–5000
Solenoid radius, cm	1
Solenoid length, cm	10
Distance of section 33 from magnet face, cm	10
Maximum magnetic field, kgauss	50–500
Particle diameter, μ	1–30
Inlet particle concentration, particles/cm^3	1×10^4–1×10^9

The following analysis is based upon a steady, one-dimensional flow approximation with given continuous distributions of velocity of the field and number density of particles. The radial gradients of the applied field will cause radial motion of particles and a redistribution in the number density of particles from one axial station to the next while the duct cross-sectional area and shape remain constant along the duct. The radial redistribution is calculated as a separate motion in which the axial component of the momentum is affected by the redistribution but the changes in the radial momentum of the fluid are entirely neglected.

Based on the previous simplifications, the equation of motion of a particle in the z direction is given by

$$m_p u_p \frac{du_p}{dz} = -3\pi\mu_f d_p(u_p - u_f) - \frac{1}{2}\frac{\pi}{8} d_p{}^3 \rho_p u_p \frac{du_p}{dz} + \frac{d_p{}^3}{4} B \frac{dB}{dz} \quad (6)$$

The first term on the right-hand side of Eq. (6) represents the Stokes drag force on the particles (calculations show that the particle Reynolds number is on the order of 1 for the parameters of this study) and the second term represents the force due to the apparent mass of the particle. The third positive term represents the magnetic driving force on the ferromagnetic particle which is assumed to have a high magnetic permeability.[3]

For an element of fluid in a control volume of length Δz, the fluid momentum equation in the z direction can be ex-

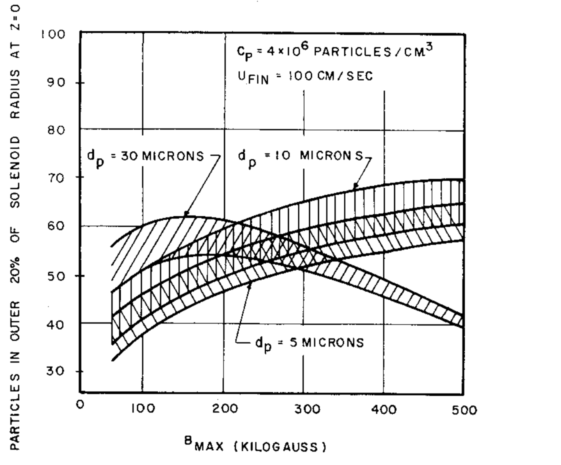

Fig. 6 Effect of initial fluid velocity, particle injection profiles, and $B_{\max}$ on particle distribution at magnet center.

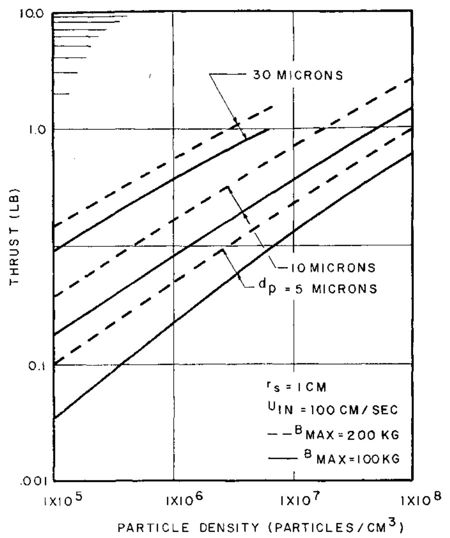

Fig. 7 Propulsor thrust as a function of particle density and diameter.

pressed as

$$\pi r_s^2 \Delta p + \int_{\Delta \text{Vol}} c_p 3\pi\mu_f d_p(u_p - u_f) d\text{Vol} = \int_{Z+\Delta Z} \rho_f u_f^2 dA - \int_Z \rho_f u_f^2 dA \quad (7)$$

In Eq. (6), the only gradient of the applied field taken into account was the axial gradient, however, as previously mentioned, the applied magnetic field has a spatial distribution which induces radial as well as axial gradients. In order to account for this radial field it was assumed that 1) the radial component of the field introduces a redistribution of the particles and 2) at the same time does not affect the motion of the fluid. This assumption was justified by comparison of the axial and radial magnetic forces on the particles for the magnet geometries of this study.

On the basis of this assumption, it is possible to study the motion of the fluid in one dimension and at the same time provide some means of accounting for the radial gradients of the applied field. If the radial component of magnetic field is included it can be shown that the radial velocity of the particle is given by

$$V_p = (d_p^2/12\pi\mu_f) B_r (dB_r/dr) \quad (8)$$

The radial magnetic field B_r and its gradient (as well as the axial magnetic field and its gradient) can be found using analytical expressions given by Callaghan and Maslen.[15]

Using the results of Eq. (8), the radial displacement of the particle while the particle moves from z to $z + \Delta z$ in the axial direction is given by

$$\Delta r = (V_p/u_p)\Delta z \quad (9)$$

The cross section of the duct can be divided into a series of radial elements. For a given distribution and total number of particles at a certain point upstream, it is possible to determine, at any subsequent axial location, the new radial distribution accounting for the shift of particles from each radial element to the next using the relation

$$N_{\text{shift}} = C_p r \Delta r \Delta z \quad (10)$$

4.3 Computed Results

Equation (6) was written in finite difference form and this equation along with Eqs. (7–10) were simultaneous solved on a CDC 3600 computer for velocity profiles of the particle and fluid as well as the pressure rise through the pumping section. Initial values for inlet fluid flow rate and velocity distribution, inlet particle distribution, geometry and magnetic field parameters were assumed. Typical assumptions and specifications used are listed in Table 2.

As previously mentioned, the results for particle concentration as a function of axial position in the magnet are needed for the design of an efficient collector system. Figure 6 gives the results of some typical calculations for this concentration at $z = 0$ with varying $B_{\max}$ and particle diameter. The range of concentrations for a given particle diameter is because of the variation of inlet particle profile distributions from linear to parabolic. It is important to note from Fig. 6 that the particle size can play an important role in the design of a collector system. As the graphical results demonstrate, there is an optimum particle size, approximately 10 μ in diameter, that gives the largest percentage of particles in the outer 20% of the solenoid radius at the magnet center. If the previously described moving belt scheme were to be used as a collection system, a more thorough study of the particle distribution as a function of axial location and particle size would be required.

Neglecting all losses in the region of the system from the particle collection point to the nozzle exit, the pressure rise through the hydromagnetic pumping section can be converted to a maximum exit velocity U_s, which would enable the system thrust to be calculated from the expression

$$\text{Thrust} = \rho_f \pi R^2 U_{\text{in}}(U_s - U_{\text{in}})/g_c \quad (11)$$

Typical results for system thrust as a function of typical system parameters are given in Figs. 7 and 8. For these

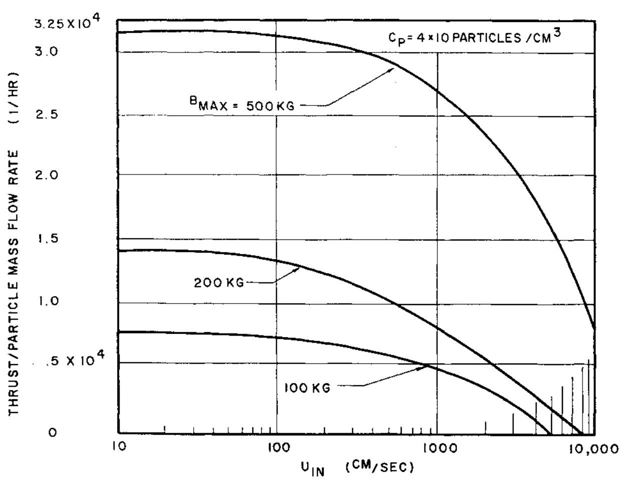

Fig. 8 Particle mass flow rate vs fluid inlet velocity.

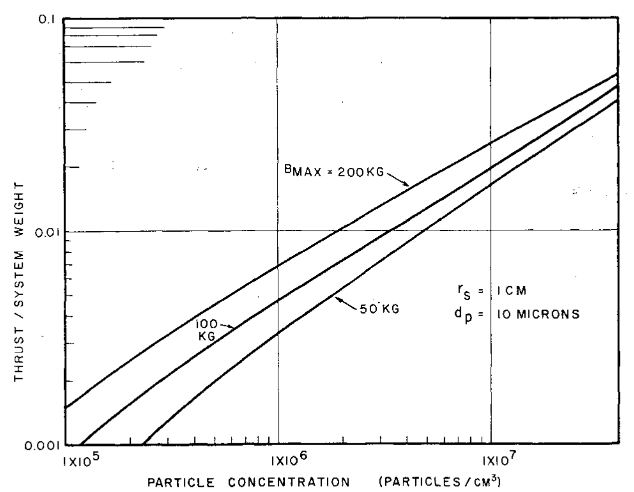

Fig. 9 Thrust to system weight ratio as a function of particle concentration and magnetic field.

calculations, it was assumed that the inlet fluid velocity profile was parabolic and that the particles were injected uniformly across the inlet. As would be expected, Fig. 7 shows that the thrust increases as the particle diameter and maximum magnetic field increase. Figure 8 illustrates effect of inlet flow rate (or centerline flow velocity) on the thrust/mass flow rate of particles for varying values of maximum magnetic field. The results illustrate that, for the particular configuration chosen, the effectiveness falls rapidly as the inlet velocity increases above values of 100 cm/sec. thus, it should be noted that as well as being a low output device, the proposed hydromagnetic propulsor is also limited to relatively low inlet velocities.

The system thrust to weight ratio can be determined with the use of the previous results for total system weight (see Fig. 4). As can be seen from Fig. 9, these values are much less than 1 for all practical values of B_{max} and particle concentration.

5. Conclusion

In this paper, the performance and system characteristics of a unique waterjet propulsion system have been presented. The low thrust output of the device and a higher thrust/particle mass flow rate at low inlet flow velocities limits the use of this device to relatively low-speed underwater craft, such as deep sea submersibles. Furthermore, the calculated low values of thrust/weight for this system, would limit the use of this device to small control thrustors instead of a primary propulsion system.

References

[1] Hatte, R. and Davis, H. J., "Selection of Hydrofoil Waterjet Propulsion Systems," *Journal of Hydronautics*, Vol. 1, No. 3, July, 1967, pp. 12–22.

[2] Arcand, L. and Comolli, C. R., "Optimization of Waterjet Propulsion for High-Speed Ships," *Journal of Hydronautics*, Vol. 2, No. 1, Jan., 1968, pp. 2–8.

[3] Hassel, W. F., "An Investigation of the Interaction of Magnetically Accelerated Colloidal Particles with Air," Ph.D. thesis, 1966, Purdue University, Lafayette, Ind.

[4] Hassel, W. F., "Atmospheric Propulsion with Magnetically Accelerated Colloids," AIAA Paper 66-926, Boston, Mass., 1966.

[5] Bitter, F., "Water-Cooled Magnets," *High Magnetic Fields*, Wiley, New York, 1961, pp. 85–100.

[6] Colyer, B., "Cryogenic Problems of Superconducting Magnetic Systems," *Proceedings of the Second International Conference on Magnet Technology*, Oxford, 1967, Rutherford Lab., Chilton, England, pp. 624–636.

[7] Lucas, E. J. and Stekly, Z. J. J., "Superconducting Coils for Special Applications," *Proceedings of the Second International Conference on Magnet Technology*, Oxford, 1967, Rutherford Lab., Chilton, England, pp. 579–587.

[8] Brown, G. V. et al., "Axial and Radial Magnetic Fields of Thick, Finite Length Solenoids," TR R-170, 1963, NASA.

[9] Wernick, J. H., "Metallurgical Aspects of High-Field Superconducting Materials," *Superconductors*, Wiley, New York, 1962, pp. 35–52.

[10] Laverick, C., "Superconducting Magnets," *Nucleonics*, Vol. 24, No. 1, Jan., 1966, pp. 46–53.

[11] Montgomery, D. B., "Generation of Intense Magnetic Field," *Journal of Applied Physics*, Vol. 36, No. 3, March, 1965, pp. 893–900.

[12] Lucas, E. J., and Stekly, Z. J. J., "Superconducting Coils for Special Applications," AMP 246, Feb., 1968, Avco Everett Research Lab., Everett, Mass.

[13] Hatch, A. M., private communication, Avco Corp., Everett, Mass., 1969.

[14] Hwang, H. U., McGowan, J. G., and Murthy, S. N. B., "Optimization of a Hydro-Magnetic Colloid Propulsion System, Part I—Motion of Colloid Particles in a Fluid Flow due to an Applied Magnetic Field," Rept. THEMIS-UM-68-1, University of Massachusetts, Amherst, Mass.

[15] Callaghan, E. E. and Maslen, S. H., "The Magnetic Field of a Finite Solenoid," TN D-465, 1960, NASA.